B.Sc. Part-I

Paper-I

Theory of Relativity

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Date 9-7-21

Expt. No. Relativistic Doppler effect for Sound and light

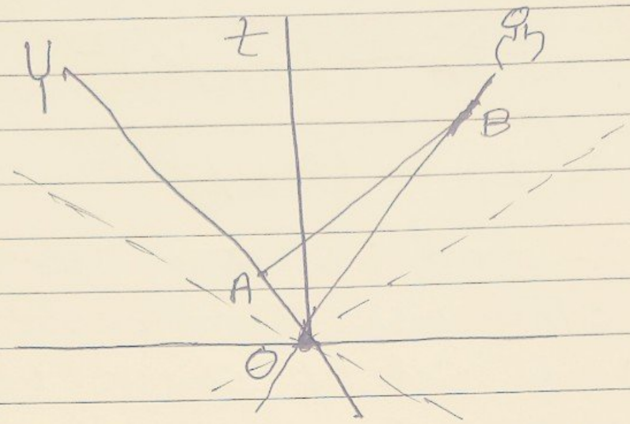

fig (9)

The relativistic Doppler shift formula is applicable to both sound and light

The traditional analysis of the Doppler effect for sound represents a low speed approximation to the exact, relativistic analysis. The fully relativistic analysis for sound is in fact equally applicable to both sound and electromagnetic phenomena.

Consider the spacetime diagram fig (9) worldlines for a tuning fork (the source) and a receiver are both illustrated on this diagram. Events O and A represent two vibrations of the tuning fork. The period of the fork is the magnitude of OA and the inverse slope of AB represents the speed of signal propagation to event B. We can therefore write

$$c_s = \frac{x_B - x_A}{t_B - t_A} \quad \text{(Speed of sound)}$$

Teacher's Signature

$v_s = -\frac{x_A}{t_A}$ $v_r = \frac{x_B}{t_B}$ (speed of source and receiver)

$|OA| = \sqrt{t_A^2 - (x_A/c)^2}$

$|OB| = \sqrt{t_B^2 - (x_B/c)^2}$

v_s and v_r are assumed to be less than c_s, since otherwise their passage through the medium will set up shock waves, invalidating the calculation. Some routine algebra gives the ratio of frequencies:

$$\frac{f_r}{f_s} = \frac{|OA|}{|OB|} = \frac{1 - v_r/c_s}{1 + v_s/c_s}\sqrt{\frac{1-(v_s/c)^2}{1-(v_r/c)^2}} \quad (9)$$

if v_r and v_s are small compared with c, the above eqn reduces to the classical Doppler formula for sound.

If the speed of signal propagation c_s approaches c, it can be shown that the absolute speeds v_s and v_r of the source and receiver merge into a single relative speed independent of any reference to a fixed medium indeed we obtain eqn (1) the formula for relativistic longitudinal Doppler shift.

Analysis of the spacetime diagram in fig (9) gave a general formula for source and receiver moving directly along their line of sight i.e. in collinear motion.

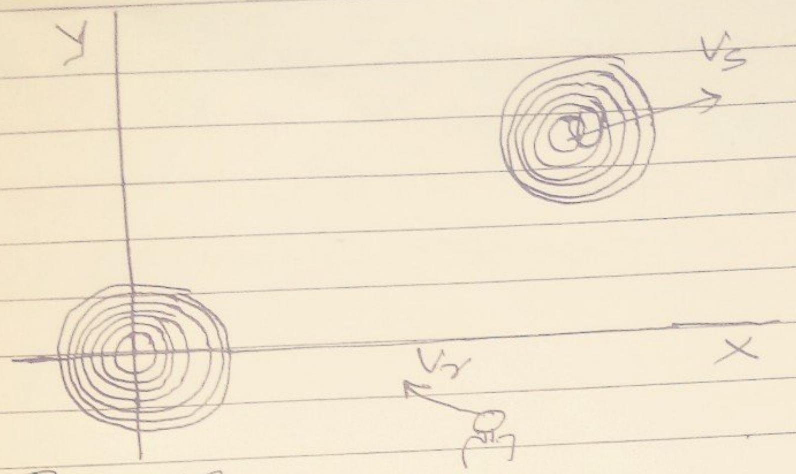

Fig — 10

A source & receiver are moving in different directions and speeds in a frame where the speed of sound is independent of direction

fig-(10) illustrates a scenario in two dimensions. The source moves with velocity V_s (at the time of emission). It emits a signal which travels at velocity c towards the receiver, which is travelling at velocity V_r at the time of reception. The analysis is performed in a coordinate system in which the signal's speed $|c|$ is independent of direction.

The ratio between the proper frequencies for the source and receiver is

$$\frac{f_r}{f_s} = \frac{1 - |V_r|/|c| \cos(\theta_c, V_r)}{1 - |V_s|/|c| \cos(\theta_c, V_s)} \sqrt{\frac{1-(V_s/c)}{1-(V_r/c)}} \quad (1)$$

The leading ratio has the form of the classical Doppler effect, while the square root term represents the relativistic correction. If we consider the angles relative to the frame the source, then $V_s = 0$ and the equation reduces to eqn (7), Einstein's 1905 formula for the Doppler effect. If we consider the angles relative to the frame of the receiver, then $v_r = 0$ and the equation reduces to eqn (6), the alternative form of the Doppler shift equation discussed previously.